**Math 231 – HW 10 Name: Troy Jeffery**

Epp 2nd Ed. 4.2 3, 6, 8

4.3 1, 2, 8, 22

**4.2 (3)** For each positive integer n, let P(n) be the formula: .

Exploration: Write out and check each of the following:

|  |  |
| --- | --- |
|  | |
| P(4): | ✔ |
| P(3): | ✔ |
| P(2): | ✔ |

Now, do the problem from the book:

|  |  |
| --- | --- |
| (a) Write out P(1). Is it true? | ✔ |
| (b) Write P(k). |  |
| (c) Write P(k+1). |  |
| (d) In a proof by mathematical induction that the formula holds for all integers n≥1, what must be shown in the inductive step? | We need to verify the formula by checking the first step (P(k)), then we assume that the formula is correct and show that it works for the next step (P(k+1)). |

**4.2 (6)** Without using Theorem 4.2.2, use mathematical induction to prove that:

for all integers n≥1.

Exploration: Write out and check each of the following:

|  |  |
| --- | --- |
|  | |
| P(4): | ✔ |
| P(3): | ✔ |
| P(2): | ✔ |

Now, do the formal proof:

|  |  |
| --- | --- |
| **Theorem:** |  |
| **Proof (By induction):** | First, we’ll verify that the initial value works.  So,      Theorem is true for n, so it should be true for n+1.  Assumed:  Proving:  Inductive Assumption  ✔ |

**4.2 (8)** Without using Theorem 4.2.3, use mathematical induction to prove that:

for all integers n≥0.

Exploration: Write out and check each of the following:

|  |  |
| --- | --- |
| P(3): | ✔ |
| P(2): | ✔ |
| P(1): | ✔ |

Now, do the formal proof:

|  |  |
| --- | --- |
| **Theorem:** | for all integers n≥0. |
| **Proof (by induction):** | Verify theorem with initial value:    ✔  If theorem is true for n, it should be true for n+1.  So, we’ll assume:  And prove:  Using Inductive Assumption:    ✔ |

**4.3 (1)** Based on the discussion of the product at the beginning of this section, conjecture a formula for general n (n≥2). Prove your conjecture by mathematical induction.

Exploration: Write out the product for each of the following, and find a simplified fraction answer:

|  |  |
| --- | --- |
| n=2: |  |
| n=3: |  |
| n=4: |  |

What do you think the correct formula is?

Now, do a formal proof that your formula is correct:

|  |  |
| --- | --- |
| **Theorem:** | for integers |
| **Proof (by induction):** | Verify theorem with initial value:  ✔  Assuming true for n, should be true for n+1.  Assumption:  Need to prove:  Using inductive Assumption:        ✔ |

**4.3 (2)** Experiment with computing values of the product , and conjecture a formula for general n (n≥1). Prove your conjecture by mathematical induction.

Exploration: Write out the product for each of the following, and find a simplified fraction answer:

|  |  |
| --- | --- |
| n=1: |  |
| n=2: | 3 |
| n=3: |  |

What do you think the correct formula is?

Now, do a formal proof that your formula is correct:

|  |  |
| --- | --- |
| **Theorem:** | for integers n 1. |
| **Proof (By induction):** | Verify theorem with first entry:    Assumption:    Proving:  Using Inductive Assumption:    ✔ |

**4.3 (8)** Prove the statement by mathematical induction:

is divisible by 3, for all integers n≥1.

Exploration: Check the statement for each of the following:

|  |  |
| --- | --- |
| n=2: | ✔ |
| n=3: |  |
| n=4: |  |

Now, do the formal proof:

|  |  |
| --- | --- |
| **Theorem:** | is divisible by 3, for all integers n≥1. |
| **Proof (by inductive):** | Verify theorem with first entry:  Assumption: is divisible by 3  i.e.  Proving:    Using inductive assumption:    Sums and products of integers are integers: |

**4.3 (22)** A sequence is defined by letting b0=5, and bk=4+bk-1, for all k≥1.

Show that bn=5+4n for all integers n≥0.

Exploration: Check the statement by showing how to calculate the value using both the recursive and explicit formulas:

|  |  |  |
| --- | --- | --- |
| *n or k value* | *answer using the recursive formula:* | *answer using the explicit formula:* |
| 0 | b0 = 5 | b0 = 5  ✔ |
| 1 | B1 = 9 | b1 = 9  ✔ |
| 2 | b2 = 13 | b2 = 13  ✔ |
| 3 | b3 = 17 | b3 = 17  ✔ |
| 4 | b4 = 21 | b4 = 21  ✔ |

Now, do the formal proof:

|  |  |
| --- | --- |
| **Theorem:** | The explicit formula bn=5+4n for all integers n≥0  Gives the same answers as  The recursive formula b0=5, and bk=4+bk-1, for all k≥1 |
| **Proof (by inductive):** | Verify theorem using initial value (n=0):  Explicit:  ✔  Recursive: ✔  Assumption for n: is equal to the recursive formula.  Proving that:  Recursive formula    ✔ |